

# New results at LHC confirming the vacuum stability and Multiple Point Principle

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## Abstract

In the present paper we argue that the correction to the Higgs mass coming from the bound state of 6 top and 6 anti-top quarks, predicted early by C.D. Froggatt and ourselves, leads to the Standard Model vacuum stability and confirms the accuracy of the multiple point principle (principle of degenerate vacua) for all experimentally valued parameters (Higgs mass, top-quark mass, etc.). Fitting to get the vacuum degeneracy requires a mass of the bound state, just in the region of the new two photon state in LHC, 750-760 GeV.

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# 1 Introduction

In this paper we are concentrated on the vacuum stability problem of the minimal Standard Model (SM) in the absence of new physics at sub-Planckian energies. The value of the Higgs mass ( $M_H = 125.66 \pm 0.34$  GeV) measured by ATLAS and CMS data [1, 2], or the more recent estimation from the combined ATLAS and CMS analysis [3, 4]:  $M_H = 125.09 \pm 0.24$  GeV, is intriguing: it is quite close to the minimum  $M_H$  value that ensures absolute vacuum stability within the SM which, in turn, implies a vanishing Higgs quartic coupling ( $\lambda$ ) around the Planck scale.

Recently LHC data show hints of new resonances having invariant masses of 300 and 750 GeV. Here, in this paper, we analyze the problem of the vacuum stability/metastability in the SM in terms of the new bosons, and try to show that these new resonances discovered by LHC can explain the exact vacuum stability. This problem is related with the concept of the Multiple Point Principle developed in Refs. [5–17], presented in the book [18] and review [19], and with the problem of tiny value of cosmological constant and dark energy (see for example Refs. [15–17, 20–25]).

Our paper is organized as follows.

Section I is an Introduction. Here we have developed a concept of the Multiple Point Principle (MPP) - theory of degenerate vacua existing in Nature.

In general, a quantum field theory allows an existence of several minima of the effective potential, which is a function of a scalar field. If all vacua, corresponding to these minima, are degenerate, having zero cosmological constants, then we can speak about the existence of a multiple critical point (MCP) at the phase diagram of theory considered for the investigation (see Refs. [5–9]). In Ref. [5] Bennett and Nielsen suggested the Multiple Point Model (MPM) of the Universe, which contains simply the SM itself up to the scale  $\sim 10^{18}$  GeV. They postulated a principle - Multiple Point Principle (MPP) for many degenerate vacua - which should solve the finetuning problem by actually making a rule for finetuning. If MPP is very accurate we may have a new law of Nature, that can help us to restrict coupling constants from a theoretical principle.

In Ref. [9] the MPP was applied (by the consideration of the two degenerate vacua in the SM) for the prediction of the top-quark and Higgs boson masses, which gave:

$$M_t = 173 \pm 5 \text{ GeV}, \quad M_H = 135 \pm 9 \text{ GeV}. \quad (1)$$

In Ref. [16] it was argued that the exact degeneracy of vacua in  $N = 1$  supergravity can shed light on the smallness of the cosmological constant. The presence of such vacua, which are degenerate to very high accuracy, may also result in small values of the quartic Higgs coupling and its beta function at the Planck scale in the phase, in which we live.

Extending the Multiple Point Principle to yet another vacuum in the pure SM, we have invented in Section 3 the idea that there exists an exceptionally light scalar bound state  $S$  constructed by 6 top + 6 anti-top quarks, and that a boson condensate of these New Bound States ( $NBS$ ) should form a third phase of the SM vacuum. According to the MPP, of course, this boson condensate should get the coupling constants finetuned so as to have a vacuum energy density (i.e. cosmological constant) very small, like the other two vacua.

Section 2 considers a question: “Could the Multiple Point Principle be exact due to corrections from the new bound state 6 top + 6 anti-top?” In previous Refs. [54–57] (see

also [58–67]), we and collaborators have speculated that 6 top + 6 anti-top quarks should be so strongly bound that the bound states would effectively function at low energies as elementary particles and can be added into loop calculations as new elementary particles in the theory, and seen, if produced, as resonances. But because of their at the end composite nature they would deviate from a fully fundamental particles by having formfactors cutting off their interactions for high four momenta. The exceptional smallness of the mass of the new bound state particle  $S$  is in fact a consequence of the degeneracy of the vacua, and thus of the Multiple Point Principle.

In Section 3 we are concentrated on the vacuum stability problem in the SM. In order to assess if the measured Higgs mass is compatible with such a peculiar condition as an absolute vacuum stability in the SM, a precise computation is needed. The study of the stability of the SM vacuum has a long history [26–47]. For energies higher than electroweak scale the analysis of vacuum stability is reduced to the study of the renormalization group evolution of the Higgs quartic coupling  $\lambda$ . The prediction (1) for the mass of the Higgs boson was improved by the calculation of the two-loop radiative corrections to the effective Higgs potential [48]. The prediction  $129 \pm 2$  GeV in Ref. [48] provided the possibility of the theoretical explanation of the value  $M_H \simeq 125.7$  GeV observed at the LHC. The authors of Ref. [49] have shown that the most interesting aspect of the measured value of  $M_H$  is its near-criticality. They extrapolated the SM parameters up to the high (Planck) energies with full 3-loop NNLO RGE precision.

It was shown in Ref. [48] that the observed Higgs mass  $M_H = 125.66 \pm 0.34$  leads to a negative value of the Higgs quartic coupling  $\lambda$  at some energy scale below the Planck scale, making the Higgs potential unstable or metastable. Since the measured value of  $M_H$  is in a window of parameters where the SM can be extrapolated till the Planck scale with no problem of consistency but also that instability could arise, a highly precise analysis for the vacuum stability becomes quite necessary. With the inclusion of the three-loop RG equations (see [49]) and two-loop matching conditions [48], the instability scale occurs at  $10^{11}$  GeV (well below the Planck scale) meaning that at that scale the effective potential starts to be unbounded from below or that a new minimum can appear, and there is a non-trivial transition probability to that minimum. According to Refs. [48, 49], the experimental value of the Higgs mass gives scenarios which are at the border between the absolute stability and metastability, the measured value of  $M_H$  puts the SM in the so-called near-critical position. Using the present experimental uncertainties on the SM parameters (mostly the top quark mass) Ref. [48] cannot conclusively establish the fate of the EW vacuum, although metastability is preferred. The above statement is the motivation for making a refined study of the vacuum stability problem.

The careful evaluation of the Higgs effective potential by Degrandi et al. [48] combined with the experimentally measured Higgs boson mass in the pure SM lead to the energy density getting negative for high values of the Higgs field. E.g. the minimum in this effective potential at the Higgs field being about  $10^{18}$  GeV would have a negative energy density, or cosmological constant, and formally the vacuum in which we live would be unstable, although it is in reality just metastable with an enormously long life time, if it is not deliberately made to decay, what would be extremely difficult. It is however only unstable vacuum with a very little margin in as far as the experimental mass of 125.7 GeV is indeed very close to the mass 129.4 GeV, which according to the calculations of Degrandi et al. [48] would make the  $10^{18}$  GeV Higgs field vacuum be degenerate with the present one. In this sense Nature has in fact chosen parameters very close to ones predicted by the “Multiple Point Principle” developed in Refs. [5–9].

Section 4 considers a model of  $6t + 6\bar{t}$  bound states. We considered the effect from the new bound states  $6t + 6\bar{t}$  on the measured Higgs mass. We considered all Feynman diagrams which give contributions of  $S$ -resonances to the renormalization group evolution of the Higgs quartic coupling  $\lambda$ . Then we calculated the main contribution of the  $S$ -resonance to  $\lambda$ , and showed that the resonance with mass  $m_S \approx 750$  GeV, having the radius  $r_0 = b/m_t$  with  $b \approx 2.35$ , gives such a positive contribution to  $\lambda$  (equal to the  $\lambda_S \approx +0.01$ ) which compensates the asymptotic value of  $\lambda \approx -0.01$ , earlier obtained by Ref. [48]. As a result, this leads to the transformation of the metastability of the EW vacuum to the stability.

Section 5 is devoted to the estimation of the radius of the new bound state  $S$  of  $6t + 6\bar{t}$ . First we reviewed the results of the mean field approximation obtained by Kuchiev, Flambaum, Shuryak and Richard.

Finally, we have considered an alternative radius estimation for the new bound state  $S$ , suggested by Froggatt and Nielsen, who have used the “eaten Higgs” exchange corrections.

Section 7 contains Summary and Conclusions. Here we present an explanation how the LHC new resonances which can be the earlier predicted bound states of 6 top and 6 anti-top quarks can provide the vacuum stability in the Standard Model confirming a high accuracy of the Multiple Point Principle.

## 2 Could the Multiple Point Principle be exact due to corrections from the new bound state 6 top + 6 anti-top?

The purpose of the present article is to estimate the corrections from the NBS 6 top + 6 anti-top to the Higgs mass 129.4 GeV predicted by Degrandi et al. [48], using the requirement of the exact MPP. This actually can be done by identifying a barely significant peak obtained at the LHC Run 2 with pp collisions at energy  $\sqrt{s} = 13$  TeV [68–70], with our bound state/resonance of 6 top and 6 anti-top. Run 2 LHC data show hints of a new resonance in the diphoton distribution at an invariant mass of 750 GeV. Since the peak, which we identify with our NBS, corresponds to a mass of 750 GeV, it means that inserting into our calculation of the correction to the predicted Higgs mass this mass of 750 GeV, we can confirm the possible vacuum stability and exact Multiple Point Principle.

### 2.1 Search for either resonance in pp collision data at $\sqrt{s} = 13$ TeV from the ATLAS detector

The Higgs boson,  $H$ , offers a rich potential for new physics searches. Recently in Refs. [68–70] the ATLAS and CMS collaborations have presented the first data obtained at the LHC Run 2 with pp collisions at energy  $\sqrt{s} = 13$  TeV. The ATLAS collaboration has  $3.2 \text{ fb}^{-1}$  of data and claims an excess in the distribution of events containing two photons, at the diphoton invariant mass  $M \approx 750$  GeV with  $3.9\sigma$  statistical significance. The ATLAS excess consists of about 14 events suggesting a best-fit width  $\Gamma$  of about 45 GeV with  $\Gamma/M \approx 0.06$ . The result is partially corroborated by the CMS collaboration

with integrated luminosity of  $2.6 \text{ fb}^{-1}$ , which has reported a mild excess of about 10  $\gamma\gamma$  events, peaked at 760 GeV. The best fit has a narrow width and a local statistical significance of  $2.6\sigma$ . Assuming a large width  $\Gamma/M \approx 0.06$  the significance decreases to  $2.0\sigma$ , corresponding to a cross section of about 6 fb.

Fig. 1(a) presents searches for a new physics in high mass diphoton events in proton-proton collisions obtained from the combination of 8 TeV and 13 TeV results. ATLAS and CMS Collaborations show a new resonance in the diphoton distribution at an invariant mass of 750-760 GeV.

Ref. [71] (see Fig. 1(b)) presents searches for resonant and non-resonant Higgs boson pair production using  $20.3 \text{ fb}^{-1}$  of proton-proton collision data at  $\sqrt{s} = 8 \text{ TeV}$  generated by the Large Hadron Collider and recorded by the ATLAS detector in 2012. A 95% confidence level upper limit is placed on the non-resonant production cross section at 2.2 pb, while the expected limit is  $1.0 \pm 0.5 \text{ pb}$ . The difference derives from a small excess of events, corresponding to  $2.4\sigma$ . In the search for a narrow resonance decaying to a pair of Higgs bosons, the expected exclusion on the production cross section falls from 1.7 pb for a resonance at 260 GeV to 0.7 pb at 500 GeV. The observed exclusion ranges from 0.7-3.5 pb. It is weaker than expected for resonances below 350 GeV. It is not excluded that the results show a resonance with mass  $\approx 300 \text{ GeV}$ .

### 3 Higgs mass and vacuum stability in the Standard Model

A theory of a single scalar field is given by the effective potential  $V_{eff}(\phi_c)$  which is a function of the classical field  $\phi_c$ . In the loop expansion  $V_{eff}$  is given by

$$V_{eff} = V^{(0)} + \sum_{n=1} V^{(n)}, \quad (2)$$

where  $V^{(0)}$  is the tree-level potential of the SM.

#### 3.1 The tree-level Higgs potential

The Higgs mechanism is the simplest mechanism leading to the spontaneous symmetry breaking of a gauge theory. In the Standard Model the breaking

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}, \quad (3)$$

achieved by the Higgs mechanism, gives masses of the gauge bosons  $W^\pm$ ,  $Z$ , the Higgs boson and fermions  $f$ .

With one Higgs doublet of  $SU(2)_L$ , we have the following tree-level Higgs potential:

$$V^{(0)} = -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \quad (4)$$

The vacuum expectation value of  $\Phi$  is:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (5)$$

where

$$v = \sqrt{\frac{m^2}{\lambda}} \approx 246 \text{ GeV}. \quad (6)$$

Introducing a four-component real field  $\phi$  by

$$\Phi^+ \Phi = \frac{1}{2} \phi^2, \quad (7)$$

where

$$\phi^2 = \sum_{i=1}^4 \phi_i^2, \quad (8)$$

we have the following tree-level potential:

$$V^{(0)} = -\frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4. \quad (9)$$

As is well-known, this tree-level potential gives the masses of the gauge bosons  $W$  and  $Z$ , fermions with flavor  $f$  and the physical Higgs boson  $H$ , which are described by the VEV parameter  $v$ :

$$M_W^2 = \frac{1}{4} g^2 v^2, \quad (10)$$

$$M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2, \quad (11)$$

$$M_f = \frac{1}{\sqrt{2}} g_f v, \quad (12)$$

$$M_H^2 = \lambda v^2, \quad (13)$$

where  $g_f$  is the Yukawa couplings of fermion with the flavor  $f$ .

### 3.2 Stability phase diagram

The vast majority of the available experimental data is consistent with the SM predictions. No sign of new physics has been detected. Until now there is no evidence for the existence of any particles other than those of the SM, or bound states composed of other particles. All accelerator physics seems to fit well with the SM, except for neutrino oscillations. These results caused a keen interest in possibility of emergence of new physics only at very high (Planck scale) energies, and generated a great attention to the problem of the vacuum stability: whether the electroweak vacuum is stable, unstable, or metastable [26–47]. A largely explored scenario assumes that new physics interactions only appear at the Planck scale  $M_{Pl} = 1.22 \cdot 10^{19} \text{ GeV}$  [50–53] and [48, 49]. According to this scenario, we need the knowledge of the Higgs effective potential  $V_{eff}(\phi)$  up to very high values of  $\phi$ . The loop corrections lead the  $V_{eff}(\phi)$  to values of  $\phi$  which are much larger than  $v$ , the location of the electroweak (EW) minimum. The effective Higgs potential develops a new minimum at  $v_2 \gg v$ . The position of the second minimum depends on the SM parameters, especially on the top and Higgs masses,  $M_t$  and  $M_H$ . It can be higher or lower than the EW one showing a stable EW vacuum (in the first case), or metastable one (in the second case). Then considering the lifetime  $\tau$  of the false vacuum (see Ref. [46]), and comparing it with the age of the universe  $T_U$ , we see that, if  $\tau$  is larger than  $T_U$ , then our universe will be sitting on the metastable false vacuum, and we deal with the scenario of metastability.

Usually the stability analysis is presented by stability diagram in the plane  $(M_H, M_t)$ . The standard results are given by Refs. [48–53], and the plot is shown in Fig. 2. For our

purposes we were guided by the “phase diagram” presented in Ref. [47]. As it was noted by authors of Ref. [47], strictly speaking, this is not a phase diagram, but this expression is still used due to the common usage in literature.

We see that the plane  $(M_H, M_t)$  is divided in Fig. 2 into three different sectors: 1) An absolute stability region (cyan), where  $V_{eff}(v) < V_{eff}(v_2)$ , 2) a metastability region (yellow), where  $V_{eff}(v_2) < V_{eff}(v)$ , but  $\tau > T_U$ , and 3) an instability (green) region, where  $V_{eff}(v_2) < V_{eff}(v)$  and  $\tau < T_U$ . The stability line separates the stability and the metastability regions, and corresponds to  $M_t$  and  $M_H$  obeying the condition  $V_{eff}(v) = V_{eff}(v_2)$ . The instability line separates the metastability and the instability regions. It corresponds to  $M_t$  and  $M_H$  for  $\tau = T_U$ .

In Fig. 2 the black dot indicates current experimental values  $M_H \simeq 125.7$  GeV [3, 4] and  $M_t \simeq 173.34$  GeV [72, 73]. It lies inside the metastability region, and could reach and even cross the stability line within  $3\sigma$ . The ellipses take into account  $1\sigma$ ,  $2\sigma$  and  $3\sigma$ , according to the current experimental errors. When the black dot sits on the stability line, then this case is named “critical”, according to the MPP concept [5–9]: the running quartic coupling  $\lambda$  and the corresponding beta-function vanish at the Planck scale:

$$\lambda(M_{Pl}) \sim 0 \quad \text{and} \quad \beta(\lambda(M_{Pl})) \sim 0. \quad (14)$$

Fig. 2 shows that the black dot, existing in the metastability region, is close to the stability line, and the “near-criticality” [49] can be considered as the most important information obtained for the Higgs boson.

The Higgs inflation scenario developed in Refs. [74–78] confirms the realization of the conditions (14).

### 3.3 Two-loop corrections to the Higgs mass from the effective potential

Still neglecting new physics interactions at the Planck scale, we can consider the Higgs effective potential  $V_{eff}(\phi)$  for large values of  $\phi$  [31] (see also [9, 12]):

$$V_{eff}(\phi) \simeq \frac{1}{4} \lambda_{eff}(\phi) \phi^4. \quad (15)$$

Here  $V_{eff}(\phi)$  is the renormalization group improved (RGE) Higgs potential [31], and  $\lambda_{eff}(\phi)$  depends on  $\phi$  as the running quartic coupling  $\lambda(\mu)$  depends on the running scale  $\mu$ . Then we have the one-loop, two-loops or three-loops expressions for  $V_{eff}$ . The corresponding up to date Next-to-Next-to-Leading-Order (NNLO) results were published in Refs. [48, 49, 79–81]. For a large range of values of  $M_H$  and  $M_t$ , the Higgs effective potential has a minimum (see also [12]). If the point  $\phi = \phi_0 = v$  is a minimum of the  $V_{eff}(\phi)$  for a given couple  $(M_H, M_t)$ , then from Eq. (15) the stability line corresponds to the conditions:

$$V_{eff}(\phi_0) = 0 \quad \text{and} \quad V'_{eff}(\phi_0) = 0. \quad (16)$$

In general, MPP predicts that:

$$V_{eff}(\phi_{min1}) = V_{eff}(\phi_{min2}) = 0, \quad (17)$$

$$V'_{eff}(\phi_{min1}) = V'_{eff}(\phi_{min2}) = 0, \quad (18)$$

where  $\phi_{min1} = v$  is the first EW vacuum, and  $\phi_{min2} = v_2$  is the second Planck scale vacuum.

The red solid line of Fig. 3 shows the running of the  $\lambda_{eff}(\phi)$  for  $M_H \simeq 125.7$  and  $M_t \simeq 171.43$ , which just corresponds to the stability line, that is, to the stable vacuum. In this case the minimum of the  $V_{eff}(\phi)$  exists at the  $\phi = \phi_0 = v \simeq 2.22 \cdot 10^{18}$  GeV, where according to (16):

$$\lambda_{eff}(\phi_0) = 0 \quad \text{and} \quad \beta(\lambda_{eff}(\phi_0)) = 0. \quad (19)$$

But as it was shown in Refs. [48,49], this case does not correspond to current experimental values.

The relation between  $\lambda$  and the Higgs mass is:

$$\lambda(\mu) = \frac{G_F}{\sqrt{2}} M_H^2 + \Delta\lambda(\mu), \quad (20)$$

where  $G_F$  is the Fermi coupling. In Eq. (20)  $\Delta\lambda(\mu)$  denotes corrections arising beyond the tree level case.

Computing  $\Delta\lambda(\mu)$  at the one loop level, using two-loop beta functions for all the SM couplings, Ref. [48] obtained the first complete NNLO evaluation of  $\Delta\lambda(\mu)$  for the two-loop QCD and Yukawa contribution to  $\Delta\lambda(\mu)$  in the SM with the electroweak gauge couplings switched off.

In Fig. 3 blue lines (thick and dashed) present the RG evolution of  $\lambda(\mu)$  for current experimental values  $M_H \simeq 125.7$  GeV [3,4] and  $M_t \simeq 173.34$  GeV [73], and for  $\alpha_s$  given by  $\pm 3\sigma$ . The thick blue line corresponds to the central value of  $\alpha_s = 0.1184$  and dashed blue lines correspond to errors of  $\alpha_s$  equal to  $\pm 0.0007$ . After the rapid variation of  $\lambda(\mu)$  around the weak scale shown in Fig. 3, these corrections play a significant role in determining the evolution of  $\lambda$  at high energies. Absolute stability of the Higgs potential is excluded by Ref. [48] at 98% C.L. for  $M_H < 126$  GeV. In Fig. 3 we see that asymptotically  $\lambda(\mu)$  does not reach zero, but approaches to the negative value:

$$\lambda \rightarrow -(0.01 \pm 0.002), \quad (21)$$

indicating the metastability of the EW vacuum. According to Ref. [48], the stability line shown in Fig. 3 by the red thick line corresponds to

$$M_H = 129.4 \pm 1.8 \text{ GeV}. \quad (22)$$

The aim of the present paper is to show that the stability line *could* correspond to the current experimental values of the SM parameters for  $M_H \simeq 125.7$  given by LHC.

## 4 The effect from the new bound states $6t + 6\bar{t}$ on the measured Higgs mass

In Refs. [54–57] there was suggested the existence of new bound states (NBS) of 6 top + 6 anti-top quarks as so strongly bound systems that they effectively function as elementary particles. Later this theory was developed in Refs. [58–67].



## 4.1 New bound states

In Ref. [54] there was first assumed that

- there exists  $1S$ -bound state  $6t + 6\bar{t}$  - scalar particle and color singlet;
- that the forces responsible for the formation of these bound states originate from the virtual exchanges of the Higgs bosons between top(anti-top)-quarks;
- that these forces are so strong that they almost compensate the mass of 12 top-anti-top quarks contained in these bound states.

The explanation of the stability of the bound state  $6t + 6\bar{t}$  is given by the Pauli principle: top-quark has two spin and three color degrees of freedom (total 6). By this reason, 6 quarks have the maximal binding energy, and 6 pairs of  $t\bar{t}$  in  $1S$ -wave state create a long lived (almost stable) colorless scalar bound state  $S$ . One could even suspect that not only this most strongly bound state  $S$  of  $6t + 6\bar{t}$ , but also some excited states exist. It is obvious that excited to a 2s or 2p state (in the atomic physics terminology), scalar and vector particles correspond to the more heavy bound states of the  $6t + 6\bar{t}$ , etc. Also there exists a new bound state  $6t + 5\bar{t}$ , which is a fermion similar to the quark of the 4th generation having quantum numbers of top-quark.

These bound states are held together by exchange of the Higgs and gluons between the top-quarks and anti-top-quarks as well as between top and top and between anti-top and anti-top. The Higgs field causes attraction between quark and quark as well as between quark and anti-quark and between anti-quark and anti-quark, so the more particles and/or anti-particles are being put together the stronger they are bound. But now for fermions as top-quarks, the Pauli principle prevents too many constituents being possible in the lowest state of a Bohr atom constructed from different top-quarks or anti-top-quarks surrounding (analogous to the electrons in an atom) the “whole system”, analogous to the nucleus in the Bohr atom. Because the quark has three color states and two spin states meaning six internal states there is in fact a shell (as in the nuclear physics) with six top quarks and similarly one for six anti-top quarks. Then we imagine that in the most strongly bound state just this shell is filled and closed for both top and anti-top. Like in nuclear physics where the closed shell nuclei are the strongest bound, we consider this NBS  $6\text{ top} + 6\text{ anti-top}$  as our favorite candidate for the most strongly bound and thus the lightest bound state  $S$ . Then we expect that our bound state  $S$  is appreciably lighter than its natural scale of 12 times the top mass, which is about 2 TeV. So the mass of our NBS  $S$  should be small compared to 2 TeV. In recent papers [57, 64] C.D. Froggatt and H.B. Nielsen estimated masses smaller than 2 TeV, but the calculation were too detailed to be trusted. It should be had in mind that such a calculation could strongly enhance the reliability of our finetuning principle - multiple point principle. From MPP it is possible to calculate the top-Yukawa coupling. Ref. [64] gave a prediction of the top Yukawa coupling:  $g_t = 1.02 \pm 14\%$ . Since the experimental top Yukawa coupling is  $g_t = 0.935$ , there is a hope that our model could be right.

In this paper, taking into account that LHC recently discovered new resonances [68–71], we analyze the problem of the vacuum stability/metastability in the SM in terms of these new bosons, and try to show that these new resonances can explain the exact vacuum stability and the exact Multiple Point Principle developed in Refs. [5–17].

## 4.2 The main diagram correcting the effective Higgs self-interaction coupling constant $\lambda$

Estimating different contributions of the bound state  $S$  (see Figs. 4(a,b)) we found that the main Feynman diagram correcting the effective Higgs self-interaction coupling constant  $\lambda(\mu)$  is the diagram of Fig. 4(a) containing the bound state  $S$  in the loop.

Now instead of Eq. (20) we have:

$$\lambda(\mu) = \frac{G_F}{\sqrt{2}} M_H^2 + \delta\lambda(\mu) + \Delta\lambda(\mu), \quad (23)$$

where the term  $\delta\lambda(\mu)$  denotes the loop corrections to the Higgs mass arising from all NBS, and the main contribution to  $\delta\lambda(\mu)$  is the term  $\lambda_S$ , which corresponds to the contribution of the diagram of Fig. 4(a):

$$\delta\lambda(\mu) = \lambda_S + \dots \quad (24)$$

The rest (see Fig. 4(b)) can be at most of a similar order of magnitude as the “dominant” one. But even if we should get a factor 2 or so it would for the mass  $m_S$  inside the quantity going into the 4th power only mean a factor the fourth root of 2, which is not so much different from 1.

We shall present the calculation of all contributions of diagrams of Fig. 4(b) in our forthcoming paper.

## 4.3 Calculation of the main diagram contribution $\lambda_S$ . Formfactor-like cut off

Taking into account that we have 12 different states of the top (or anti-top) quark in the bound state  $S$  (top-quark or anti-top-quark with 3 colors and 2 spin-1/2 states), we obtain the following integral corresponding to the diagram of Fig. 4(a):

$$\lambda_S \simeq G_{HSS}^4 \int \frac{d^4 q}{(2\pi)^4} \frac{\mathfrak{F}_0(q^2) \mathfrak{F}_0((q+p_1)^2) \mathfrak{F}_0((q+p_1-p_3)^2) \mathfrak{F}_0((q-p_2)^2)}{(q^2 - m_S^2) ((q+p_1)^2 - m_S^2) ((q+p_1-p_3)^2 - m_S^2) ((q-p_2)^2 - m_S^2)}, \quad (25)$$

where  $q$  is the loop 4-momentum, and  $p_i$  ( $i = 1, 2, 3, 4$ ) are the external 4-momenta of the Higgs bosons. Here  $p_1 + p_2 = p_3 + p_4$ . The coupling constant between the three scalar particles SSH (see [82]) is:

$$G_{HSS} = 24m_S \frac{g_t}{\sqrt{2}}. \quad (26)$$

Eq. (25) contains formfactors  $\mathfrak{F}_0(q_i^2)$ , which describe formfactor-like cut off.

If NBS has a very small radius then in loop calculations we can treat such a bound state as a “fundamental” particle. This means that we introduce more Feynman rules with the bound state corresponding propagators or some appropriate effective coupling vertices. But if we have a bound state with an extension, or the radius,  $r_0$ , which is not sufficiently small, then we expect to have in loops formfactors behaving like

$$\mathfrak{F}_0(q^2) = \exp\left(\frac{1}{6} \langle r^2 \rangle q^2\right), \quad (27)$$

where  $q$  is the four momentum relevant for the effective vertex or propagator in question. After the Wick rotation the loop four-momentum  $q$  will be space-like so that  $q^2 < 0$ . Thus, when we want to estimate, what correction we need to include for a given loop with an aim to improve a contribution of bound states running around the loop, then we suppose that the exponential formfactor (27), once for each propagator, comes in multiplying the integrand of the Feynman diagram which damps the contribution of the diagram numerically.

In Eq. (27) we have for our  $\lambda_S$ -loop calculation:

$$\langle \bar{r}^2 \rangle = 3r_0^2. \quad (28)$$

Let us present now the radius of the bound state  $S$  as

$$r_0 = \frac{b}{m_t}. \quad (29)$$

If  $r_0$  is the radius  $a_B$  of the Bohr Hydrogen-atom-like bound state  $S$  containing 12 top-quarks and having  $Z = 11/2$ , then  $r_0 = a_B$ .

The next Sections are devoted to the correct estimation of the radius of the new bound state  $S$  of  $6t + 6\bar{t}$  in details, and then we compare it with the radius needed for exact MPP.

Let us continue now the calculation of the contribution  $\lambda_S$  given by diagram of Fig. 4(a).

Neglecting 4-momenta  $p_i$  ( $i = 1, 2, 3, 4$ ) in the integrand of Eq. (25) as very small, we obtain:

$$\lambda_S \approx G_{HSS}^4 \int \frac{d^4 q}{(2\pi)^4} \frac{(\mathfrak{F}_0(q^2))^4}{(q^2 - m_S^2)^4}. \quad (30)$$

Considering the Wick rotation, we obtain the loop four-momentum  $q_E$  which is space-like and  $q^2 = -q_E^2$ . Then the formfactor (27) is:

$$\mathfrak{F}_0(q_E^2) = \exp\left(-\frac{1}{2}r_0^2 q_E^2\right), \quad (31)$$

Using the expression (31) for Eq. (30), we obtain:

$$\begin{aligned} \lambda_S &\approx \frac{G_{HSS}^4}{(2\pi)^4} \int d^4 q \frac{(\exp(\frac{1}{2}r_0^2 q^2))^4}{(q^2 - m_S^2)^4} = \frac{G_{HSS}^4}{(2\pi)^4} \int d^4 q_E \frac{\exp(-2r_0^2 q_E^2)}{(q_E^2 + m_S^2)^4} \\ &= \frac{G_{HSS}^4}{16\pi^2} \int_0^\infty q_E^2 dq_E^2 \frac{\exp(-2r_0^2 q_E^2)}{(q_E^2 + m_S^2)^4} = \frac{G_{HSS}^4}{16\pi^2 m_S^8} \int_0^\infty q_E^2 dq_E^2 \frac{\exp(-2r_0^2 q_E^2)}{(q_E^2/m_S^2 + 1)^4} \\ &= \left(\frac{G_{HSS}}{2m_S}\right)^4 \frac{1}{\pi^2} \int_0^\infty y dy \frac{\exp(-(2r_0^2 m_S^2) y)}{(y + 1)^4} \\ &\approx \frac{1}{4\pi^2} \left(\frac{G_{HSS}}{r_0 m_S^2}\right)^4 = \frac{1}{\pi^2} \left(\frac{6g_t}{b} \cdot \frac{m_t}{m_S}\right)^4, \end{aligned} \quad (32)$$

where  $y = q_E^2/m_S^2$  and  $G_{HSS}$  is given by Eq. (26).

If any  $S$ -resonance (with  $m_S \approx 300$  GeV or 750 GeV) gives

$$\lambda_S \simeq 0.01, \quad (33)$$

then this contribution transforms the metastable (blue) curve of Fig. 3 into stable (red) curve, and we have exact vacuum stability and exact MPP.

From Eq. (32) we have:

$$\lambda_S \simeq \frac{1}{\pi^2} \left( \frac{6g_t \xi}{b} \right)^4, \quad (34)$$

where

$$\xi = \frac{m_t}{m_S}.$$

Vacuum stability corresponds to the condition:

$$\left( \frac{\xi}{b} \right)^4 \simeq 0.01 \frac{\pi^2}{(6g_t)^4} \approx 0.00008. \quad (35)$$

or

$$\frac{\xi}{b} \simeq 0.095, \quad (36)$$

what gives:

$$b \simeq \frac{\xi}{0.095}. \quad (37)$$

We see that  $\xi_1 \approx 173/750 \approx 0.231$  for the resonance with the mass  $m_S \approx 750$  GeV and  $\xi_2 \approx 173/300 \approx 0.577$  for  $m_S \approx 300$  GeV. Very likely one of the two mentioned resonances could turn out to be a statistical fluctuation. Therefore, if only the resonance with mass 750 GeV provides the vacuum stability, then its radius  $r_0 = b/m_t$  has to have:

$$b = b_1 \simeq \frac{0.231}{0.095} \approx 2.34. \quad (38)$$

But in the case of the 300 GeV resonance (only), the vacuum stability is possible for

$$b = b_2 \simeq \frac{0.577}{0.095} \approx 6.07. \quad (39)$$

If both resonances with different  $b'_i$  ( $i = 1, 2$ ) give their contribution to  $\lambda_S$ , then we must in order to get the exact MPP have:

$$\left( \frac{\xi_1}{b'_1} \right)^4 + \left( \frac{\xi_2}{b'_2} \right)^4 \simeq 0.01 \frac{\pi^2}{(6g_t)^4} \approx 0.00008. \quad (40)$$

In Section 6 we obtained  $b \simeq 2.34$  (see below), for the experimental values  $g_t \approx 0.935$  and  $m_S \simeq 750$  GeV, it gives:

$$\lambda_S \simeq 0.009, \quad (41)$$

what transforms the metastable (blue) curve of Fig. 3 into stable (red) curve. Of course, the uncertainty coming from the contributions of diagrams shown in Fig. 4(b) can reach 25%, and then we have:

$$\lambda_S \simeq 0.009 \pm 0.002. \quad (42)$$

It is not excluded that the mentioned resonance with the mass  $m_S \approx 300$  GeV is not a bound state  $6t + 6\bar{t}$ , but only a statistical fluctuation.

It is very important to investigate experimentally the radii of the discovered resonances.

## 5 Estimation of the radius of the new bound state $S$ of $6t + 6\bar{t}$ in the mean field approximation

The kinetic energy term of the Higgs field and the top-quark Yukawa interaction are given by the following Lagrangian density:

$$L = \frac{1}{2} \mathcal{D}_\mu \Phi_H \mathcal{D}^\mu \Phi_H + \frac{g_t}{\sqrt{2}} \bar{\psi}_{tR} \psi_{tL} \Phi_H. \quad (43)$$

According to the Salam-Weinberg theory, the top-quark mass  $M_t$  and the Higgs mass  $M_H$  are given by relations (12) and (13) with  $v \approx 246$  GeV.

Let us imagine now that the NBS is a bubble in the EW-vacuum and contains  $N$  top-like constituents. It is known that inside the bubble (bag) the Higgs field can modify its VEV. Implications related with this phenomenon have been discussed in Refs. [55, 56] and Refs. [83–85]. Then inside bound state  $S$  the VEV of the Higgs field is smaller than  $v$ :

$$v_0 = \langle 0 | \Phi_H | 0 \rangle, \quad \text{where} \quad \frac{v_0}{v} < 1, \quad (44)$$

and the effective Higgs mass inside the bubble/bag are smaller than the corresponding experimental masses:  $m_h = (v_0/v) M_H$ . In this case the attraction between the two top (or anti-top) quarks is presented by the Yukawa type of potential:

$$V(r) = -\frac{g_t^2/2}{4\pi r} \exp(-m_h r). \quad (45)$$

Assuming that the radius  $r_0$  of the bubble is small:

$$m_h r_0 \ll 1, \quad (46)$$

we obtain the Coulomb-like potential:

$$V(r) = -\frac{g_t^2/2}{4\pi r}. \quad (47)$$

The attraction between any pairs  $tt$ ,  $t\bar{t}$ ,  $\bar{t}\bar{t}$  is described by the same potential (47). By analogy with Bohr Hydrogen-atom-like model, the binding energy of a single top-quark relatively to the nucleus containing  $Z = N/2$  top-quarks have been estimated in Refs. [54–57]. The total potential energy for the NBS with  $N = 12$  has  $Z = 11/2$  (see explanation in Ref. [57]) and is:

$$V(r) = -\frac{11}{2} \frac{g_t^2/2}{4\pi r}. \quad (48)$$

Here we would like to comment that the value of the mass  $m_h$ , which belongs to the Higgs field inside the NBS  $6t + 6\bar{t}$ , can just coincide with estimates given by Refs. [86–89]. The results:  $\mathbf{max}(m_h) = 29$  GeV and  $\mathbf{max}(m_h) = 31$  GeV correspond to Ref. [87] and Ref. [89], respectively.

In Refs. [54–57] it was suggested that the Higgs boson, responsible for generating the masses of fermions in the Standard model, couples more strongly to the heavy quarks (top, beauty, etc.,) than to the light ones. The question has been raised whether the attraction mediated by Higgs exchange could produce new type of bound states. If the Higgs boson coupling to top quarks is large enough to generate a whole spectroscopy of new bound states (NBS), then for large enough number of quarks this binding energy can be strong enough to stabilize top quarks, as bound neutrons are stabilized in nuclei. The

particular state discussed in [54–57] is for the number of quarks  $N = 12$ , 6 t-quarks and 6 anti-t-quarks, with all spin and color values, which all occupy the same lowest 1S orbital. In a simple approximation the binding energy for this state is so large that the total mass of this 1S bound state turns to be zero (with claimed accuracy). At least, the mass of any NBS is much smaller than the mass of 12 top-quarks. Refs. [54–57] considered proposals of how to find such states at Tevatron and LHC.

In the Standard model the interaction of t-quarks with the Higgs boson is proportional to the large mass of the t-quark,  $M_t = 173.34$  GeV [72], with a coupling  $g_t = 0.935 \pm 0.008$  [72]. Therefore the effective Coulomb coupling is about as strong as a QCD coupling constant  $\alpha_s$  at the same scale, with the additional advantage that quarks and anti-quarks of any color are equally attracted by the Higgs exchange. Thus the binding energy should grow with the number of quarks.

The calculations of Refs. [54–57] gave only a rough estimate for the binding energy, which was based on analogy with Bohr energies of the Hydrogen atom. However, the Bohr atom has an attractive Coulomb center, while for multi-top bound states theory considers the charge which is smeared out over the volume of the multi-top bound state. The calculations of Refs. [86–89] show that this leads to the huge difference for 12 t-quarks, which are not deeply bound at all, but rather unbound, if realistic limits on the Higgs mass is used.

The authors of Refs. [86–89] have used first the variational approach, and then a self-consistent mean-field approach. The many-body and recoil corrections are expected to be small,  $\simeq 1/N$ . The number of t-quarks  $N = 12$  gives the accuracy of the binding energy  $\simeq 10\%$ . For weak coupling the non-relativistic approximation is valid. Then the interaction between top quarks due to Higgs boson exchange may be described by the following Hamiltonian (in units  $\hbar = c = 1$ ):

$$H = \sum_{i=1}^N \frac{\vec{p}_i^2}{2m_t} + \sum_{i<j} V(\vec{r}_{ij}), \quad (49)$$

where  $r_{ij}$  are the interquark distances, and

$$V(r) = -\frac{\alpha_H}{r} \exp(-m_h r). \quad (50)$$

Here  $m_h$  is the Higgs mass inside the bound state, and the coefficient  $\alpha_H$  is introduced for the strength of the Higgs boson exchange. The t-channel exchange in the top-quark scattering leads to the effective Coulomb coupling  $\alpha_H = g_t^2/(8\pi)$ . The inclusion of s- and u- channels makes the effective interaction between t-quarks more strong (see [57]). In general:

$$\alpha_H = \frac{(g_t)_{eff}^2}{8\pi} = \frac{\kappa g_t^2}{8\pi}. \quad (51)$$

The authors of Refs. [86, 89] presumed that  $\kappa \approx 2$  and  $\alpha_H \approx g_t^2/(4\pi)$ . Considering  $g_t \approx 1$ , they obtained  $\alpha_H \approx 1/(4\pi) \approx 0.08$ , and used this value of  $\alpha_H$  in their calculations. For a preliminary investigation of the existence of new type of bound states due to the Higgs exchanges, the authors of Refs. [86, 89] have used the Hamiltonian (49) and examine its spectral properties. If  $N$ , the number of constituents, does not exceed 6 top quarks and 6 top anti-quarks, the colour and spin degrees of freedom can endorse the constraints of anti-symmetrisation, and for the orbital variables, the Hamiltonian (49) can be considered as acting on effective bosons. This is a reason why our attention has been focused on the  $6t + 6\bar{t}$  system, which in Ref. [89] was named “dodecatoplet”, by analogy with a “pentaquark”.

The level energies  $E_N$  of the Hamiltonian  $H$ , and in particular its ground-state, are given by the following scaling way:

$$E_N = (m_t, \alpha_H, m_h) = \frac{m_h}{m_t} \epsilon_N(G), \quad (52)$$

where

$$G = \frac{m_t}{m_h} \alpha_H. \quad (53)$$

The variational approach assumes that the wave function of the multi-top system incorporates a product of the  $N$  orbitals. The ground-state energy  $\epsilon_2 = E_2$  has the variational upper bound:

$$\tilde{\epsilon}_2 = \min_a [t(a) - Gp(a)], \quad (54)$$

corresponding to a single normalized Gaussian:

$$\psi_a(r) \propto \exp\left(-\frac{a}{2}r^2\right), \quad (55)$$

where the range parameter  $a$  (dimensionless in units  $m_h = 1$ ) is optimized.

The radial wave function  $\phi_a(r) = r\psi_a(r)$  gives the dimensionless radius of the bound state:

$$\tilde{r}_0 = \frac{1}{2\sqrt{a}} = b \frac{m_h}{m_t}. \quad (56)$$

In Eq. (54) we have functions of the radius of toponium - the bound state with  $N=2$ :

$$t(a) = \frac{3a}{2}, \quad \text{and} \quad p(a) = 2\sqrt{\frac{a}{\pi}} - \exp\left(\frac{1}{4a}\right) \operatorname{erfc}\left(\frac{1}{2\sqrt{a}}\right). \quad (57)$$

For  $N$  constituents the variational energy is:

$$\tilde{\epsilon}_N = \min_a (N-1) [t(a) - NGp(a)]. \quad (58)$$

Constructing a function:

$$\frac{\tilde{\epsilon}_N}{N-1} = \min_a [t(a) - NGp(a)] = \min_a F(z), \quad (59)$$

where  $F(z)$  depends on the variable  $z = 1/(2\sqrt{a}) = \tilde{r}_0$ , and using functions (57), we obtain for  $N = 12$  the following expression:

$$F(z) = t(a) - 6Gp(a) = \frac{3}{8z^2} - \frac{6G}{\sqrt{\pi}z} + 6Ge^{z^2} \operatorname{erfc}(z). \quad (60)$$

Minimization of Eq. (60) gives the following equation:

$$\frac{1}{2G(z_0)} = \frac{4}{\sqrt{\pi}}z_0 - \frac{8}{\sqrt{\pi}}z_0^3 + 8z_0^4 e^{z_0^2} \operatorname{erfc}(z_0) = f(z_0), \quad (61)$$

where

$$f(z) = \frac{4}{\sqrt{\pi}}z - \frac{8}{\sqrt{\pi}}z^3 + 8z^4 e^{z^2} \operatorname{erfc}(z). \quad (62)$$

Here  $z_0$  is a position of the minimum of the binding energy, where  $F'(z)|_{z=z_0} = 0$ . The function (62) has a maximum at  $1/2G \approx 1.19$  and  $z_0 \approx 1.3$ . According to Eq. (53), we have a maximal value of  $m_h$ :  $m_h(\text{max}) = 31 \text{ GeV}$  [89] for  $\alpha_H \approx 0.075$ , what is close to  $\kappa = 2$  and  $g_t = 1$  in Eq. (51) [89]. The result  $\max m_h = 29 \text{ GeV}$  was obtained in Ref. [86]. We see that 12 top-quarks cannot bind, if the Higgs effective mass inside the

bound state is larger than  $m_h(\text{max})$ : binding energy per quark for  $S$ ,  $\epsilon_{12}/11$ , is negative for  $G > G_{\min} = 1/2f(z_0) \approx 0.42$  (compare with the result of Ref. [89]).

In Eqs. (60) and (61) we have:

$$G = \gamma\alpha_H \quad \text{and} \quad z = \frac{b}{\gamma}, \quad (63)$$

where  $\gamma = m_t/m_h$ , and

$$\gamma_{\min} \approx \frac{2G_{\min}}{2\alpha_H} = \frac{1}{2.38\alpha_H} \approx 5.6 \quad (64)$$

for  $\alpha_H \approx 0.075$ . As a result, we have:

$$b(\text{max } m_h) \approx 1.3\gamma_{\min} \approx 1.3 \cdot 5.6 = 7.28. \quad (65)$$

This is a result for  $b$ , if the effective Higgs mass inside the bound state is maximal:  $m_h(\text{max}) \approx 31$  GeV [89]. However, we do not believe that  $m_h$  inside the bound state is large.

For  $m_h = 0$  ( $z = 0$ ) we have obtained (from (62) and (63)) the following result:

$$\frac{1}{b\alpha_H} = \frac{f(z)}{z} \Big|_{z=0} = \frac{8}{\sqrt{\pi}}, \quad (66)$$

and

$$b \approx 2.95. \quad (67)$$

Below, in contrast to the mean field approximation, developed in Refs. [86–89], we give an alternative estimation of radius of the bound state  $S$ .

## 6 An alternative radius estimation for the new bound state $S$

Here we have an aim to compute the radius of the bound state  $S$  by an alternative way presumably described in Ref. [57].

### 6.1 The “eaten Higgs” exchange corrections

Now we want, contrary to what was considered by Froggatt and Nielsen in Ref. [57], to take into account the “eaten Higgs”- corrections which we called “local”. Really we follow the thoughts of [57] that the inclusion of the “eaten Higgs” exchange leads to an effect as if the  $g_t$  had in first approximation a factor  $4^{1/4}$  (then  $\kappa = 2$  in Eq. (51)). But by more careful estimation, we rather argued that instead of just this factor  $4^{1/4} = \sqrt{2}$ , we should use  $\sqrt{2} \exp(-4.0\%) = 1.359$ . Now because of the existence of the gluon contribution included into the definition of  $\alpha_H$ , this  $\alpha_H$  is not quite increased by a factor  $\kappa \approx 1.359^2 \approx 1.85$  due to such an effect. Rather we might take into account that for the binding energy in the Bohr atom approximation, instead of  $(1/2)\alpha_H^2$ , which is  $\propto g_t^4$  in the case of ignoring of gluon interactions, we have an effective power  $g_t^{(2.9)}$ . This 2.9 is estimated by saying that the gluon term in the  $\alpha_H$  makes up about 27.4% of the total  $1.83 + 4g_t^2 = 1.83 + 4.84$  (see [57]). Thus, the Higgs part of  $\alpha_H$  is 72.6 %, and an effective power dependence is



$(1.359g_t)^{(2.9)} = 2.434g_t^{(2.9)}$ , what means that the binding energy is changed by a factor  $1.359^{(2.90)} = 2.434$ .

Now the philosophy of the “eaten Higgs” correction is implemented by a change in the binding energy without it coming from “long distance” calculation by Richard [89]. The effect of the simply changing the binding energy is in writing the binding energy per quark as  $\epsilon_{(12)}/11 = Am_t/m_h$  and considering the  $A$  being increased by the factor 2.434. Thus, if for the purpose of achieving the “light” mass (much lighter than  $12M_t \approx 2$  TeV) for the bound state  $S$ , we need  $A = 0.49$ , then we should require that *before making the “eaten Higgs” correction* the value of  $A$  should be decreased by a factor 2.434, that is, we should require that an estimate of the binding energy ignoring the “eaten Higgs” interaction should only lead to the binding energy  $\epsilon_{(12)}/11 = A_{lbe}m_t/m_h$ , where  $A_{lbe}$  is:

$$A_{lbe} = A_{(\text{before adding local binding energy})} = \frac{0.49}{2.434} = 0.201. \quad (68)$$

## 6.2 Froggatt-Nielsen corrections to the $S$ -radius estimates

In Ref. [57] Froggatt and Nielsen estimated the radius of the bound state  $S$ , using the assumption that the binding energy per quark is given by the value  $Am_t = m_t/2$ . Here  $A = 0.5$  is needed for obtaining the light mass for the bound state  $S$ . But now formally instead of  $A = 0.5$  we shall use  $A_{lbe} = 0.201$  given by Eq. (68). With this new value of the assumed binding energy we shall follow the virial theorem which was used previously in Ref. [57] for  $S$ -radius estimates.

This means that if we, for example, imagine that a given quark (or anti-quark) is so close to another one that the potential between them dominate and is proportional to  $1/r$  (here we ignore the Higgs mass, considering its correction later), then from the virial theorem we would have that in the ground state of this two-quark system the average potential energy  $\langle V \rangle$  is given by the following equality:

$$\langle V \rangle = -2 \langle T \rangle, \quad (69)$$

where  $\langle T \rangle$  is the average kinetic energy. Then the binding energy  $\epsilon_{(12)}/11$ , apart from the sign, coincides with just the kinetic energy:

$$\epsilon_{(12)}/11 = \langle V \rangle - \langle T \rangle = -\langle T \rangle. \quad (70)$$

In general, we have:

$$\langle \vec{p}^2 \rangle \approx 2m_t (\text{eff.rel.}) \langle T \rangle, \quad (71)$$

where  $m_t (\text{eff.rel.})$  is the effective relativistic mass of the top-quark inside the bound state. Of course, this relation is true at least for transverse (extra) momenta. Here the “eff” means that the rest top-quark mass is not to be given as the usual  $m_t$  but it is obtained by an average Higgs field value in the interior of the bound state.

In the previous article [57] it was assumed (see again the discussion of this problem below) that as an approximation, the Heisenberg uncertainty relation  $\sigma_x \sigma_{p_x} \geq 1/2$  has *equality* rather than *inequality*. That is, we assume that the wave functions can be approximated by the following relations:

$$\langle x^2 \rangle \langle p_x^2 \rangle = 1/4, \quad (72)$$

$$\langle \vec{r}^2 \rangle \langle \vec{p}^2 \rangle = 9/4. \quad (73)$$

If these relations indeed were true, then we would get:

$$\langle \vec{r}^2 \rangle = \frac{9}{4 \langle \vec{p}^2 \rangle} = \frac{9}{8m_t \text{ (eff.rel.)} \langle T \rangle} = \frac{9}{8m_t \text{ (eff.rel.)} |A|m_t} = \frac{9}{8(1 - |A|)|A|m_t^2}. \quad (74)$$

This means (in the notations used) that

$$\langle \vec{r}^2 \rangle = 3r_0^2, \quad (75)$$

$$r_0 = \frac{b}{m_t}, \quad (76)$$

and we obtain:

$$b = \sqrt{\frac{\langle \vec{r}^2 \rangle}{3}} m_t = \frac{3}{8(1 - |A|)|A|} = \frac{3}{8(1 - 0.201)0.201} = 2.34, \quad (77)$$

when we have inserted for  $|A|$  the value 0.201, taking into account the eaten Higgs correction (68).

## 7 Summary and Conclusion

In this paper we assumed that recently discovered at the LHC new resonances with masses  $m_S \approx 300$  GeV and 750 GeV, or just one of them, are new scalar  $S$  bound states  $6t + 6\bar{t}$ , earlier predicted by C.D. Froggatt and authors. It was shown that these NBS can provide the vacuum stability and exact accuracy of the Multiple Point Principle, according to which the two vacua (existing at the Electroweak and Planck scales) are degenerate.

We discussed the possibility of different new bound states (LHC resonances) to give the correction to the Higgs mass coming from the bound states of 6 top and 6 anti-top quarks. We showed that the value of their radii are essential for the transformation of the metastable SM vacuum into the stable one.

We calculated the main contribution of the  $S$ -resonance to the renormalization group evolution of the Higgs quartic coupling  $\lambda$ , and showed that the resonance with mass  $m_S \approx 750$  GeV, having the radius  $r_0 = b/m_t$  with  $b \approx 2.34$ , gives the positive contribution to  $\lambda$ , equal to the  $\lambda_S \approx +0.01$ . This contribution compensates the asymptotic value of the  $\lambda \approx -0.01$ , which was earlier obtained in Ref. [48], and therefore transforms the metastability of the EW vacuum into the stability.

We have considered the calculation of the NBS radius in the model of the mean field approximation, which was developed by authors of Refs. [86–89]. But then we considered an alternative way of the radius calculation for the  $S$  bound state, developed in the Froggatt-Nielsen relativistic model. The last model gave successful results for our aims to get the SM vacuum stability and an almost exact effect of the MPP.

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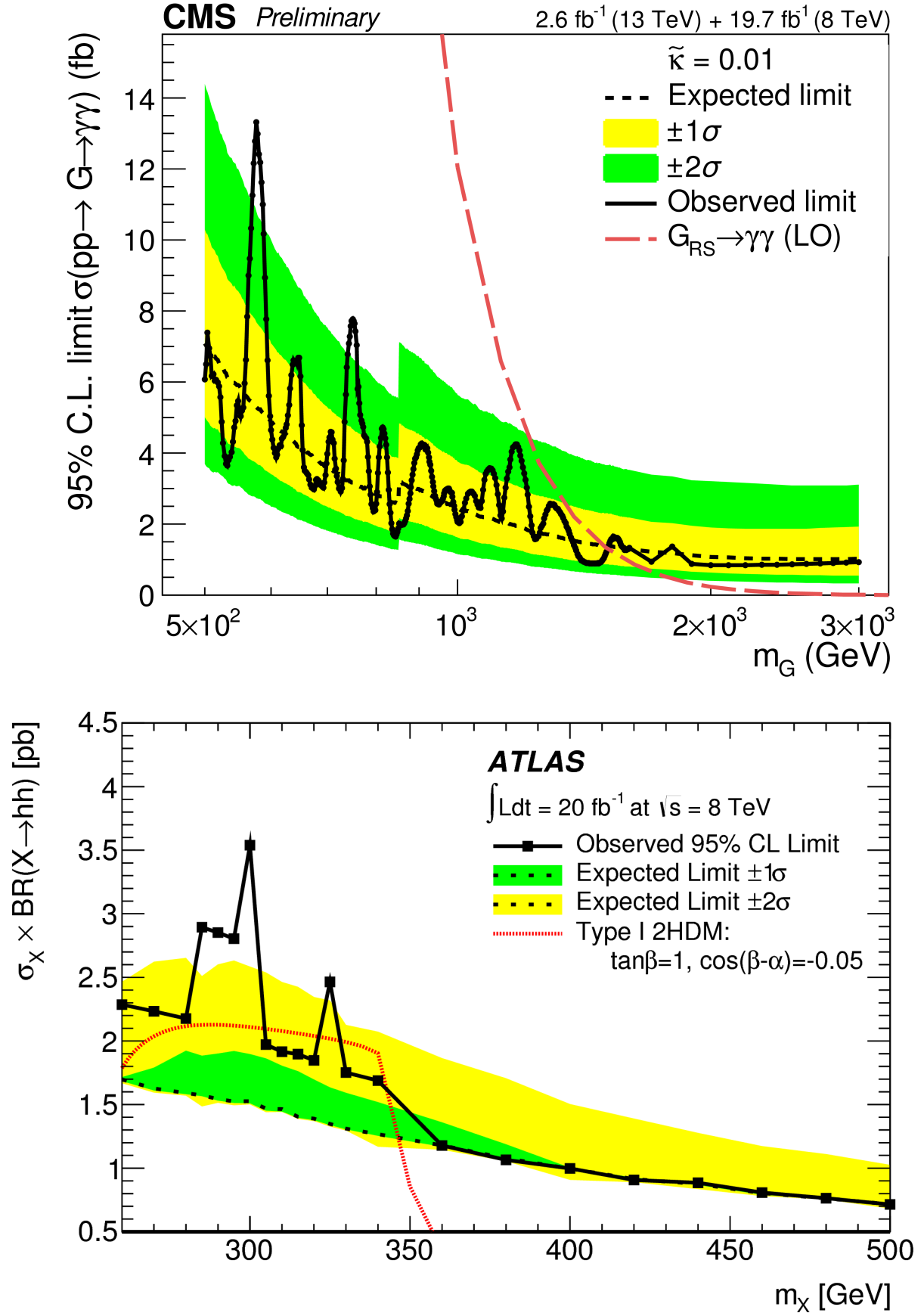


Fig. 1: The first figure presents searches for a new physics in high mass diphoton events in proton-proton collisions at 13 TeV and 8 TeV combined analysis. ATLAS and CMS Collaborations show a new resonance in the diphoton distribution at an invariant mass of 750-760 GeV. The next figure presents searches for resonant and non-resonant Higgs boson pair production using  $20.3 \text{ fb}^{-1}$  of proton-proton collision data at  $\sqrt{s} = 8 \text{ TeV}$  generated by the LHC and recorded by the ATLAS detector in 2012. The results show a resonance with mass  $\approx 300 \text{ GeV}$



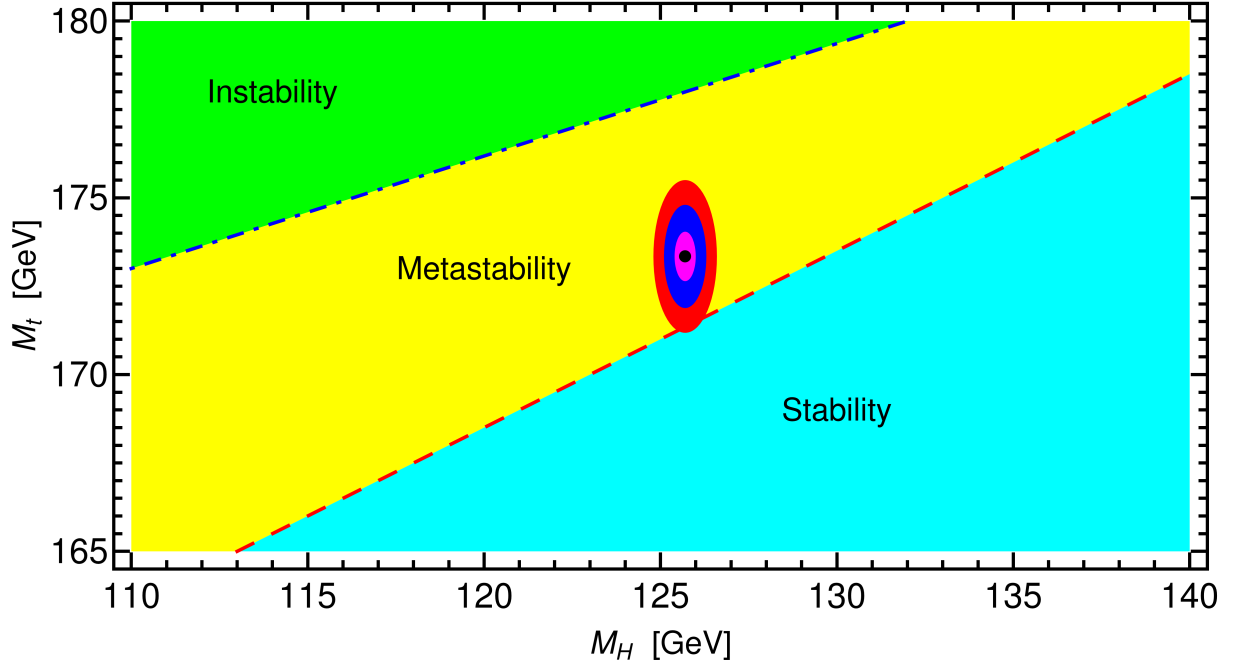


Fig. 2: The stability phase diagram obtained according to the standard analysis. The  $(M_H, M_t)$  plane is divided in three sectors: absolute stability, metastability and instability regions. The dot indicates current experimental values  $M_H \simeq 125.7$  GeV and  $M_t \simeq 173.34$  GeV. The ellipses take into account  $1\sigma$ ,  $2\sigma$  and  $3\sigma$ , according to the current experimental errors.

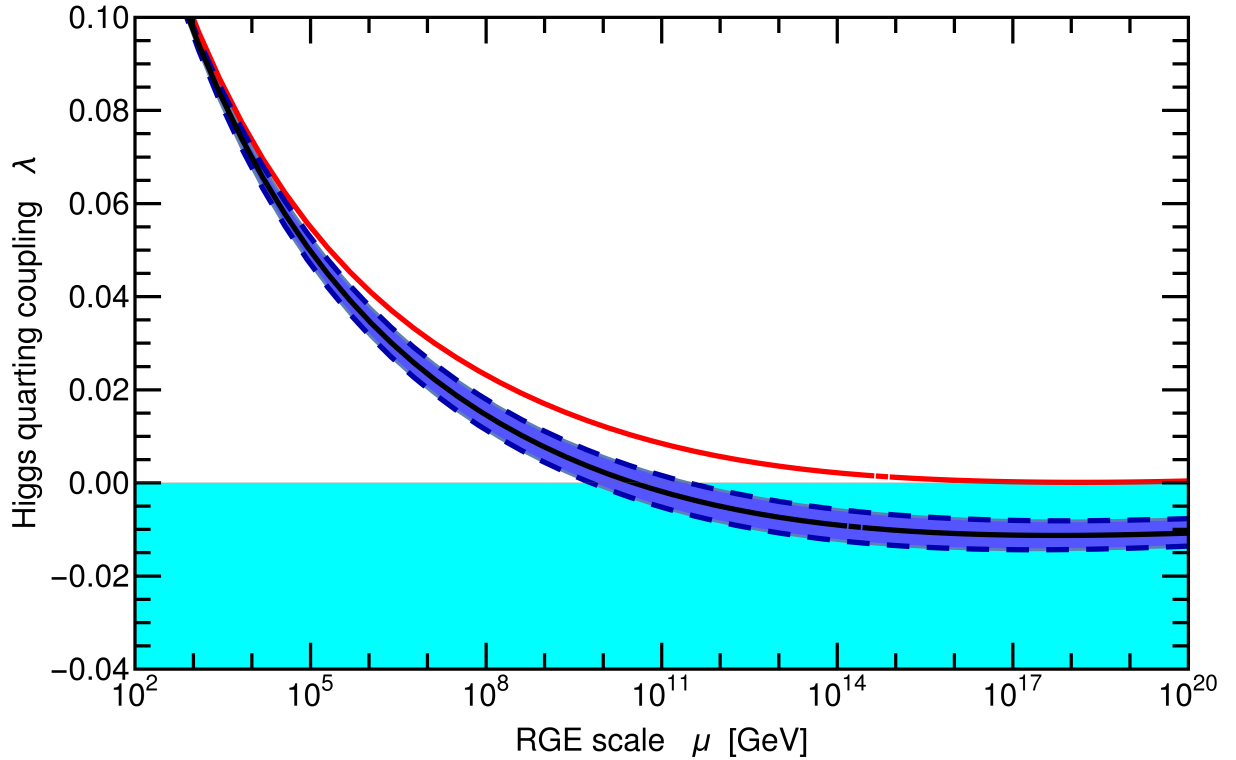


Fig. 3: The RG evolution of the Higgs selfcoupling  $\lambda$  for  $M_t \simeq 173.34$  GeV and  $\alpha_s = 0.1184$  given by  $\pm 3\sigma$ . Blue lines present metastability for current experimental data, red (thick) line corresponds to the stability of the EW vacuum

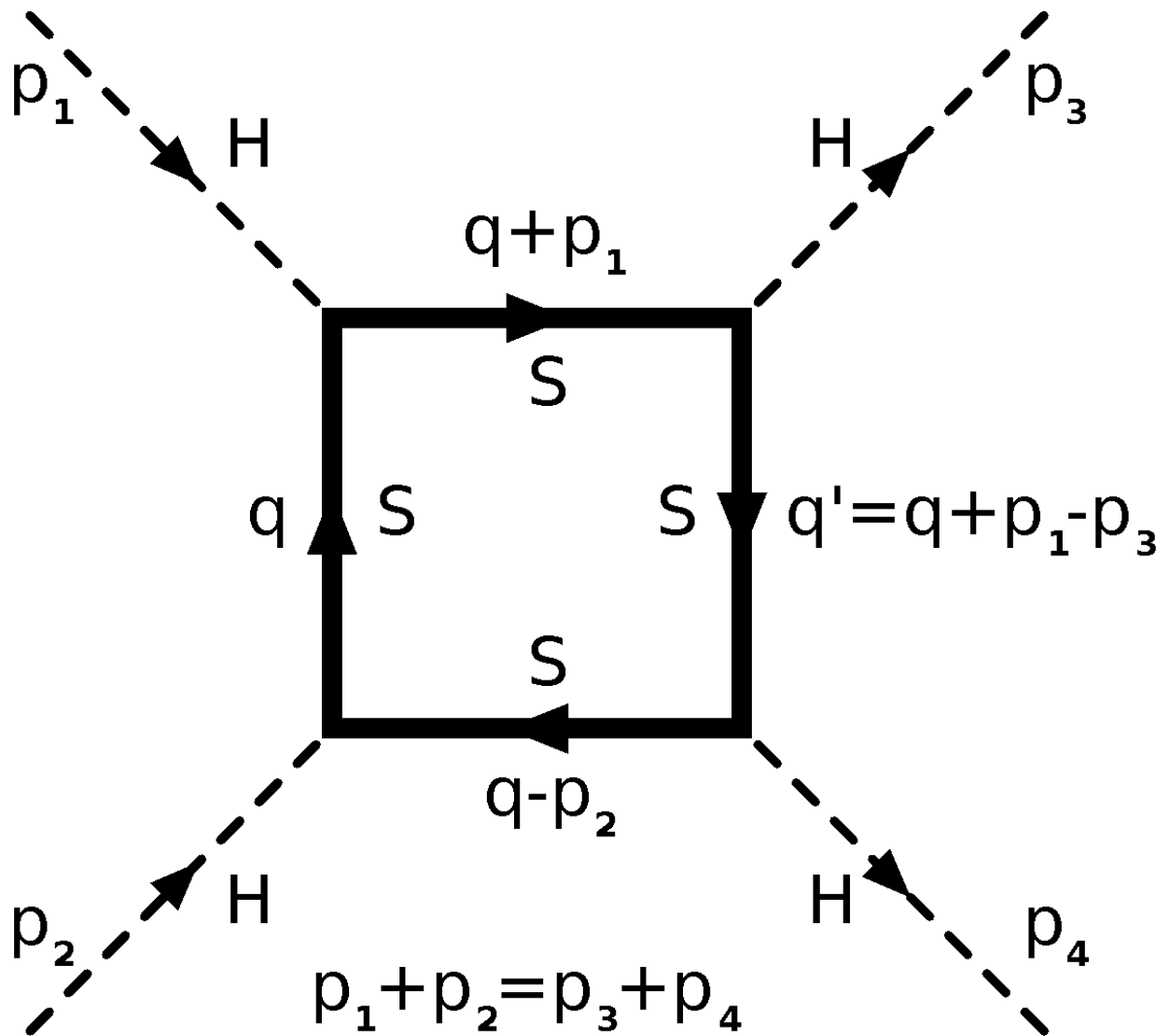


Fig. 4: (a) The Feynman diagram corresponding to the main contribution of the  $S$  bound state  $6t + 6\bar{t}$  to the running Higgs selfcoupling  $\lambda$ .

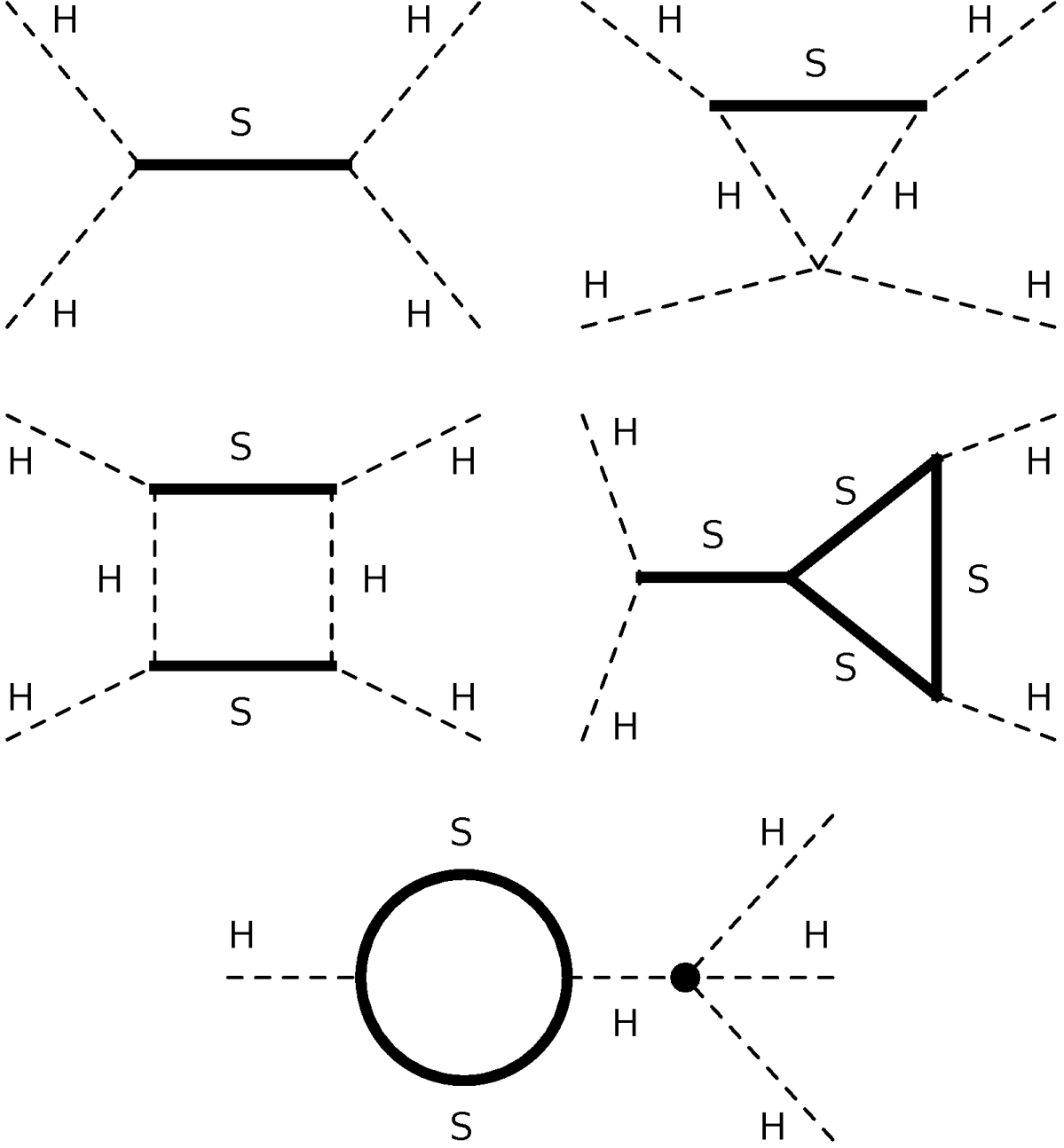


Fig. 4: (b) Feynman diagrams of other contributions of NBS to the  $\lambda_S$ , which are smaller within 20-25%.